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**Lecture 15: Isocosts and Optimality**

1. Isocosts
   1. Just as isoquants are the production version of indifference curves, isocosts are the production version of budget constraints.
   2. An *isocost* *line* shows all the combinations of labor and capital which can be purchased for a given total cost.
2. Capital
   1. Usually, we think of buying capital all at once, say $150 million for a new airplane. But this approach doesn’t quite capture how firms actually buy capital nor does it capture how capital is used in our analysis.
      1. For example, you might buy the plane in one year but use it for thirty years.
   2. To adjust for this, we can *amortize* capital across its life by dividing its life span by its cost (life span/cost). The result is the *depreciation* (the period depends on the unit life span is in).
      1. For example, the $150 million plane for thirty years has a depreciation of $5 million per year. Even though the company spends $150 million all at once, it really only costs $5 million per year since the firm can resell the plane at a discount.
   3. We also must consider opportunity cost of capital. When you buy capital outright, you could have put some of the money that you haven’t spent yet (i.e. the money that you could get back if you sold the capital input) in an investment. High interest rates imply capital is more expensive.
   4. The *user cost of capital* is thus economic depreciation + interest rate times value of capital
   5. We can convert that into a rate per current dollar of capital (which, based on our calculations, will be constant across periods) such that:

*r/$= depreciation rate + interest rate*

* + 1. In our airplane example, the rate of depreciation is 1/30 (0.0333). If our interest rate is 0.05, then r is 0.0833.
  1. Firms also rent capital (e.g. office space) at a *rental rate* but we don’t need to consider the interest or depreciation rates there. Why not? Because the rental rate will be priced to account for them.
     1. The seller of the rented capital faces the same choices the firm does: continue renting or liquidate and invest. Their rental rate must consider depreciation and interest rates.
     2. Thus *r* should be the same whether the capital is rented or bought. Rental rate equals the user cost of capital.
     3. Note that *r* considers the value of each unit of capital; *r/$* does not.

1. Isocost formula
   1. Consider the following equation for the cost of production:

C = wL + rK

Where w is our wage rate, or the cost of labor.

* 1. By isolating as we did a budget constraint we get:

K = C/r – (w/r)L

1. Optimal bundle
   1. Once again, we find the optimal bundle of inputs where the isocost curve is tangent to the isoquant curve, or where

MRTS = -ΔK/ΔL = MPL / MPK = w / r

* 1. Rewriting, we can claim that at this point,

MPL / w = MPK / r

* 1. With several different isoquants and isoquants, you can construct an *expansion path*—curve passing though optimal bundles of inputs at varying levels of production.
     1. This is found by shifting the isocost curve out or in (as if the total budget for production is changing, rather than just one input).