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**Lecture 09: Bayes Rule**

1. Testing for rarity.
   1. Suppose you’re interested which of your employees are on drugs. This is very rare, of course, but if they work with heavy machinery or sensitive information it’s particularly important that they’re clean.
   2. Suppose national surveys reveal that **one out of every 100 people** who work in your industry use drugs.
   3. The problem is that no drug test is 100% accurate. Still, most can get close. Suppose you use a test which is 90% accurate, or **its sensitivity and specificity are 90%**:
      1. *Sensitivity*—describes the ability to detect a positive state. For every 10 drug users, 9 will get a (correct) positive test result. Subtracting sensitivity from one tells you the chance of getting a false negative.
      2. *Specificity*— describes the ability to a detect negative state. For every 10 non-users, 9 will get a (correct) negative test result. Subtracting specificity from one tells you the chance of getting a false positive.
      3. For simplicity, I’m assuming the sensitivity and specificity are the same but it doesn’t have to be that way.
      4. In practice, there is often a trade-off between the two. A test that is made to be very sensitive often means its specificity decreases. For example, a sensitive metal detector will not only pick up more threats (true positives) but also detect more false positives such as cell phones and belt buckles.
   4. Let’s begin by making a *truth table*, or a table of probabilities with all possible outcomes. Here there are two variables—the drug use of the subject and the result of the test—each with two outcomes—“drug user” or “not drug user” and “positive” or “negative.”

|  |  |  |
| --- | --- | --- |
| *Subject* | *Test Result* | *Pr()* |
| User | Positive |  |
| User | Negative |  |
| Not User | Positive |  |
| Not User | Negative |  |

* + 1. For each combination, what is the probability that combination will occur? (Remember how we calculate probabilities!)
  1. We can now determine how likely it is for someone—regardless of actual drug use—to test positive. How do we do that?
  2. Finally, what is the likelihood that a positive test indicates you are a user?

1. Bayes Rule
   1. Before testing, you had a 1% chance of being a user and a 99% of not being a user. Note that, after testing, a positive result increased your likelihood of being a user and decreased your likelihood of not being a user.
   2. This is an example of *Bayes Rule*—a way to describe how one should adjust their beliefs to account for evidence.
   3. To better understand Bayes, let’s first understand some terminology.
      1. *Pr(A)*—the probability event A will occur.
      2. *Pr(A|B)*—the probability event A will occur assuming event B happens.
      3. *~* —this symbol means “not.” Pr(~A) means “the probability event A will not occur.”
      4. Employing the example from Part II, let’s let **U** be “employee is a user” (so ~U is “employee is not a user”) and **P** be “the test came back positive for drugs” (so ~P is “the test is not positive, or came back negative for drugs).
   4. Bayes’ Theorem is as stated:
      1. Note why the denominators are equivalent. If A happens, how likely is B? If A doesn’t happen, how likely is B? Adjusting for the likelihood A happens, and you get the probability that B happens. This last step is similar to how we solved the Simpson Paradox; it’s a weighted average.
   5. We want to know “what is the probability that someone who tested positive for drugs is actually a user?” Or, given that the test is positive, how likely is it that they actually use drugs? Or, what is Pr(U|P)?
      1. Note the result is as before: 8.3%. Only 8.3% of positive results are actually drug users!
   6. Bayes Rule thus tells of two interrelated things:
      1. When you receive new information (such as the results of a drug test), it tells you how much to adjust your estimation of the truth.
      2. It reminds you that because no test is 100% accurate, its results should not be weighed too heavily, especially if it is testing for something rare (since the false positives will overwhelm the true positives).
2. One more example
   1. Consider a hypothetical computer program which checks computers for files relating to terrorist activity. Being “flagged” by the program makes the owner of the computer a target for future investigations.
      1. Assume 1 in 10,000 people (p=0.0001) are connected to terrorist activities.
      2. The sensitivity is 99.9%.
      3. The specificity is 98.0%.
   2. What percent of computers flagged by the program are not actually connected with terrorist activities?
      1. In other words, how often will authorities investigate an innocent person?
   3. What percent of computers not flagged by the program will be actually connected with terrorist activities?
      1. In other words, how often will authorities ignore a genuine suspect?