David Youngberg

Econ 304—Bethany College

**Lecture 14: Mixed Strategies**

1. Pure strategies
   1. So far, we’ve been talking in terms of *pure strategy*—a nonrandom course of action. In the Prisoner’s Dilemma, the players will always Confess//Confess. In the Chicken Game, the players will either always Swerve//Straight or Straight//Swerve. No player will choose based on chance.
   2. This system works fine when there’s always at least one Nash Equilibrium, but what happens if there is no Nash Equilibrium? Consider the game below, where a cop is chasing a robber and each must decide where to run to. If they choose the same thing, the cop catches the robber. If they choose differently, the robber gets away (though the robber has a slightly better chance to get away in the building).

|  |  |  |  |
| --- | --- | --- | --- |
|  | | **Robber** | |
| *Alleyway* | *Building* |
| **Cop** | *Alleyway* | 2 , -2 | -3 , 3 |
| *Building* | -2 , 2 | 1 , -1 |

Though there is no Nash equilibrium we know that if they were to play this game many times the goal would be unpredictability. Choosing the same strategy over and over ensures you will lose.

1. Mixed strategies
   1. The question then becomes “how often will each chooses one or the other?” This is what we call the *mixed strategy*—choosing randomly between pure strategies with specific probabilities.
      1. A mixed strategy is about keeping the other person guessing. If the cop knew where the robber was going before the cop chose, the robber would lose every time, and vice versa.
      2. In mixed strategy, actual value—not simply the relative value—matters.
   2. Suppose the cop will choose Alleyway with a probability of *p* and Building with a probability of *1 – p*. Call this the cop’s p-mix strategy.
      1. Below, we expand our table to include the p-mix strategy. Note these are expected values.

|  |  |  |  |
| --- | --- | --- | --- |
|  | | **Robber** | |
| *Alleyway* | *Building* |
| **Cop** | *Alleyway* | 2 , -2 | -3 , 3 |
| *Building* | -2 , 2 | 1 , -1 |
| *p-mix* | 2p + -2(1 – p),  -2p + 2(1 – p) | -3p + 1(1 – p),  3p + -1(1 – p) |

* 1. Looking only at the first numbers for the cop, we can construct a graph with *p* on the x-axis and expected payoff on the y-axis.
     1. There will be two lines, one for each of the robber’s strategies.

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

Expected Payoff

Cop’s p-mix

1

-3

-2

2

Robber Plays *Alleyway*

Robber Plays *Building*

* + 1. Because this is a zero-sum game, when the cop wins, the robber loses an equal amount. Thus the robber will favor a strategy that causes the cop to get the least amount, given a particularly p-mix. With that in mind, we can re-imagine the diagram:

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

Expected Payoff

Cop’s p-mix

-3

-2

Robber Plays *Alleyway*

Robber Plays *Building*

* 1. Note that where the two intersect is the cop’s maximin, or the highest of his minimum values.
     1. Mathematically, it is where the two strategy lines equal so that the robber is indifferent between both strategies.
     2. 2p + -2(1 – p) = -3p + 1(1 – p)
     3. 4p – 2 = 1 – 4p
     4. p = 3/8; the cop will choose *Alleyway* 3/8 of the time. His expected value is -0.5. Note this is higher than the cop’s maximin strategy of -2.
  2. We can do the same thing with the robber: at what probability will the robber choose *Alleyway*?
     1. Let *q* be the probability that the robber will choose *Alleyway*.
     2. -2q + 3(1 – q) is the robber’s payoff if the cop chooses *Alleyway* and 2q – 1(1 – q) is the robber’s payoff if the cop chooses *Building*.

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9

Expected Payoff

Robber’s q-mix

-2

-1

Cop Plays *Alleyway*

Cop Plays *Building*

* + 1. Mathematically, -2q + 3(1 – q) = 2q – 1(1 – q), or q = 1/2. Unlike the cop, the robber should flip a coin to determine where he goes even though going to the building might be considered “better.” The robber’s expected value is also 0.5 which is higher than the robber’s maximin payoff of -1. Note as well that the robber’s expected payoff is the positive version of the cop’s. Not surprising since this is a zero sum game.

1. Mixed strategies in Equilibrium
   1. We know that at a p-mix of 3/8, the cop will get the same expected payoff regardless if the robber chooses *Alleyway* or *Building*.
   2. We also know that at a q-mix of 0.5, the robber will get the same expected payoff regardless if the cop chooses *Alleyway* or *Building*.
   3. But if either deviates from their mix, the other will want to go one way or the other all the time.
      1. The left illustrates that the Cop will choose Building (p=0) if the Robber leans towards Building (q<½) and will choose Alleyway (p=1) if the Robber leans towards Alleyway (q>½).
      2. The right illustrates that the Robber will choose Alleyway (q=1) if the Cop leans Building (p<3/8) and the Robber will choose Building (q=0) if the Cop leans Alleyway (p>3/8).

*p*

*q*

1

1

0

Cop

*p*

*q*

1

1

0

Robber

* 1. Combining them we see an intersection: the equilibrium.

*p*

*q*

1

1

0

* + 1. This is called the *prevent-exploitation method* because it’s built around making sure the other player doesn’t take advantage of your decisions.