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**Lecture 21: A Location Model**

1. Monopolistic competition
	1. We now take monopolistic competition one step further with a location model, allowing not just heterogeneous products, but also heterogeneous consumers.
	2. “Location” can mean two different things here. One is the “distance” between what the product is and what the consumer wants. A more intuitive interpretation, however, is a physical distance.
		1. Imagine a beach, with consumers along the beach and an ice cream stand along the beach. Different consumers must travel different distances (different transportation costs) to buy ice cream.
	3. If you could determine the cost to travel, would you want it to be high (so it’s hard to move away from your stand and thus charge a high price), low (so it’s easy to move toward your stand and thus get a large quantity), or some mixture of the two?
2. Linear approach
	1. To simplify, we will take a linear approach (one metric of distance) as described by Hotelling (1929).
		1. The distance is L (L>0)
		2. Consumers are uniformly distributed, one per point along L. Thus, there are L consumers, with *x* being the name of a consumer (located *x* away from the origin).
		3. It cost τ to travel one unit of distance.
		4. Assume two firms, A and B, with A to the left of B. Below is a diagram to illustrate.

A

B

0

L

xi

a

L-b

b

a

* + 1. What are the prices and quantities charged under this model?
	1. Now we consider the utility of consumer xi if she buys from A…

$$U\_{x}=-p\_{A}-τ\left|x-a\right|$$

* 1. And if she buys from B…

$$U\_{x}=-p\_{B}-τ\left|x-(L-b)\right|$$

* 1. When is xi indifferent between A and B?

$$-p\_{B}-τ\left|x-(L-b)\right|=-p\_{A}-τ\left|x-a\right|$$

$$x=\frac{p\_{B}-p\_{A}}{2τ}+\frac{L-b+a}{2}$$

* + 1. Note that since all x’s closer to A will consume A’s product, x is also the quantity of product A sells. In other words, this is A’s demand function.
		2. To make B’s demand function, we simply determine how many buyers remain, or L – x, and adjust the other demand curve to match (multiply by -1 and then add L).

$$L-x=\frac{p\_{A}-p\_{B}}{2τ}+\frac{L+b-a}{2}$$

1. Bertrand Equilibrium
	1. In Bertrand Equilibrium, firms set price. We use the same technique here. Firm A takes pB as given and chooses pA.
		1. We multiply the demand curve by pA to get revenue.
		2. Note no costs were assumed in this model. Maximizing revenue is the same as maximizing profits.

$$π\_{A}=\frac{p\_{A}p\_{B}-p\_{A}^{2}}{2τ}+\frac{p\_{A}\left(L-b+a\right)}{2}$$

$$\frac{∂π\_{A}}{∂p\_{A}}=\frac{p\_{B}-2p\_{A}}{2τ}+\frac{L-b+a}{2}=0$$

$$\frac{p\_{B}-2p\_{A}}{τ}+L-b+a=0$$

$$2p\_{A}-p\_{B}=τ\left(L-b+a\right)$$

$$p\_{A}=\frac{p\_{B}+τ\left(L-b+a\right)}{2}$$

* + 1. And then we do the same for Firm B.

$$π\_{B}=\frac{p\_{B}p\_{A}-p\_{B}^{2}}{2τ}+\frac{p\_{B}\left(L+b-a\right)}{2}$$

$$\frac{∂π\_{B}}{∂p\_{B}}=\frac{p\_{A}-2p\_{B}}{2τ}+\frac{L+b-a}{2}=0$$

$$\frac{p\_{A}-2p\_{B}}{τ}+L-a+b=0$$

$$2p\_{B}-p\_{A}=τ\left(L-a+b\right)$$

$$p\_{B}=\frac{p\_{A}+τ\left(L-a+b\right)}{2}$$

* 1. Now we combine them to find price…

$$p\_{A}=\frac{\frac{p\_{A}+τ\left(L-a+b\right)}{2}+τ\left(L-b+a\right)}{2}$$

$$p\_{A}=\frac{p\_{A}+τ\left(L-a+b\right)}{4}+\frac{τ\left(L-b+a\right)}{2}$$

$$\frac{3p\_{A}}{4}=τ\left[\frac{\left(L-a+b\right)}{4}+\frac{\left(L-b+a\right)}{2}\right]$$

$$\frac{3p\_{A}}{4}=\frac{τ}{4}\left[\left(L-a+b\right)+2\left(L-b+a\right)\right]$$

$$3p\_{A}=τ\left(3L+a-b\right)$$

$$p\_{A}=\frac{τ}{3}\left(3L+a-b\right)$$

$$p\_{B}=\frac{τ}{3}\left(3L+b-a\right)$$

* 1. And we use the price to determine quantity…

$$Q\_{A}=\frac{\frac{τ}{3}\left(3L+b-a\right)-\frac{τ}{3}\left(3L+a-b\right)}{2τ}+\frac{L-b+a}{2}$$

$$Q\_{A}=\frac{τ\left[\left(3L+b-a\right)-\left(3L+a-b\right)\right]}{6τ}+\frac{3\left(L-b+a\right)}{6}$$

$$Q\_{A}=\frac{\left(3L+b-a\right)-\left(3L+a-b\right)+3\left(L-b+a\right)}{6}$$

$$Q\_{A}=\frac{3L+b-a-3L-a+b+3L-3b+3a}{6}$$

$$Q\_{A}=\frac{3L-b+a}{6}$$

$$Q\_{B}=\frac{3L-a+b}{6}$$

* 1. Finally, we combine them for profit…

$$π\_{A}=\frac{τ}{18}\left(3L+a-b\right)^{2}$$

$$π\_{B}=\frac{τ}{18}\left(3L+b-a\right)^{2}$$

* + 1. Note that we’ve unambiguously answered our question: you want high transportation costs.
		2. We see in practice. For example, firms build brand identity to make it costly for customers to switch products.
1. Firms choosing location
	1. What if firms could choose their location? Would Firm A increase its profits if it moves closer to Firm B?
		1. That can easily be answered given the profit function and a little math. How?
	2. This implies that firms will want to be located in the same spot. That means they have no differentiation.
		1. Or, L = a + b and thus pA = pB = 0