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**Lecture 17: Cost Minimization**

1. Production curves
	1. Like indifference curves, we use production curves which are downward sloping and convex.
	2. The general form of such a function F(K,L) is the maximum output that can be made with given values of inputs, or F(K,L) = Q.
2. Lagrangian
	1. Once again, we set up a Lagrangian. However, we do in a different way: we are interested in minimizing costs, not maximizing utility.
	2. To construct it, we set up the Lagrangian, θ, as so:

$$θ=wL+rK-λ(F\left(K,L\right)-Q)$$

with λ as the “Lagrangian multiplier.”

* + 1. Note that our production curve is set equal to zero.
	1. To determine the maximization result, we take the derivative with respect to X, Y, and λ. Then set equal to zero. Then we solve.

$$\frac{∂θ}{∂L}=w-λMP\_{L}\left(K,L\right)=0$$

$$\frac{∂θ}{∂K}=r-λMP\_{K}\left(K,L\right)=0$$

$$\frac{∂θ}{∂λ}=-F\left(K,L\right)+Q=0$$

* 1. We can then rewrite the results as:

$$w=λMP\_{L}\left(K,L\right)$$

$$r=λMP\_{K}\left(K,L\right)$$

$$F\left(K,L\right)=Q$$

* + 1. This last equation is not very interesting; the other two are because we can isolate lambda and…

$$λ=\frac{MP\_{L}\left(K,L\right)}{w}=\frac{MP\_{K}\left(K,L\right)}{r}$$

$$\frac{MP\_{L}\left(K,L\right)}{MP\_{K}\left(K,L\right)}=\frac{w}{r}$$

* 1. It’s worth noting that this is the same result as if we did constrained maximization, as we did with consumer choice theory.
		1. Cost minimization is choosing the lowest isocost for a given isoquant.
		2. Production maximization is choosing the highest isoquant for a given isocost.
1. Cobb-Douglas production function
	1. The standard production equation is the *Cobb-Douglas production function*—a form where Q = AKαLβ; K is capital, L is labor, and A is technology. Most of the time α, β < 1 (allowing for diminishing marginal returns).
	2. If α + β > 1, there are increasing returns to scale.
	3. If α + β = 1, there are constant returns to scale.
	4. If α + β < 1, there are decreasing returns to scale.

$$θ=wL+rK-λ(AK^{α}L^{β}-Q)$$

$$rβAK^{α}L^{β-1}=wαAK^{α-1}L^{β}$$

$$L=\frac{βrK}{wα}$$

$$\frac{AK^{α}β^{β}r^{β}K^{β}}{α^{β}w^{β}}=Q$$

$$K^{α+β}=\frac{\left(\frac{αw}{βr}\right)^{β}Q}{A}$$

$$K=\left(\frac{αw}{βr}\right)^{^{β}/\_{α+β}}\left(\frac{Q}{A}\right)^{^{1}/\_{α+β}}$$

$$L=\left(\frac{βr}{αw}\right)^{^{α}/\_{α+β}}\left(\frac{Q}{A}\right)^{^{1}/\_{α+β}}$$