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LECTURE 29: PROBABILITY

I. Probability

a. Express probability as a decimal or fraction; the likelihood something will occur is never greater than 1.

   i. 20% becomes 0.2

   ii. 1% becomes 0.01

   iii. 0.5% becomes 0.005

b. For notation, P(A) means the probability that A will occur.

c. To know the probability of two things happening at the same time (x **AND** y), you multiply probabilities. This makes sense: "and" means something should be less likely and multiplying values less than one makes the chance of it happening decrease.

   i. The chance of having breakfast and having sunshine should be less than the chance of just having breakfast and it should be less than the chance of just having sunshine.

   ii. The equation is: $P(A \text{ and } B) = P(A)*P(B)$

   iii. This assumes the events are independent, meaning the outcome of one event doesn't affect the likelihood of the other event. Sunshine doesn't make it more or less likely you'll have breakfast.

   iv. If it does change the chance, we'll have to use a tree diagram, which we'll discuss later in this lecture.

d. To know the chance of two possible things happening (x **OR** y), you add probabilities. This makes sense: "or" means something should be more likely and adding makes the chance of it happening increase.

   i. The equation is: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

      1. Why subtract P(A and B)? Because we have to make sure we don't double count an event.

      2. Suppose the chance of sunshine is 0.8 and the chance you'll eat breakfast is 0.9. The chance of sunshine or eating breakfast can't be 1.7—that's more than 100%! You want to find the chance of breakfast and add on the chance of sunshine with no breakfast. (Or the chance of sunshine and add on the chance of breakfast without sunshine.)

	Sunshine (0.8)	No Sunshine (0.2)
Breakfast (0.9)	0.8*0.9 = 0.72	0.2*0.9 = 0.18
No Breakfast (0.1)	0.8*0.1 = 0.08	0.2*0.1 = 0.02

3. Note the answer is 0.72 + 0.08 + 0.18 = 0.98; the only result that would not fulfill the sunshine or breakfast requirement is no sunshine and no breakfast.
4. Or: 0.8+0.9 – 0.8*0.9 = 1.7 – 0.72 = 0.98
5. Without subtraction P(A and B), you count P(A and B) twice:

WHOOPS!

	Sunshine (0.8)	No Sunshine (0.2)
Breakfast (0.9)	0.8*0.9 = 0.72	0.2*0.9 = 0.18
No Breakfast (0.1)	0.8*0.1 = 0.08	0.2*0.1 = 0.02

6. To put it another way, imagine 100 days. 80 of them have sunshine. 90 of them have breakfast. If you're tallying up every day that has one of each, you can't do 80+90 because 72 of those days have both. You're counting those days twice so you subtract 72 to account for the fact that 72 of your 90 breakfast days already have sunshine or 72 of your 80 sunshine days already have breakfast.

ii. If the probabilities are mutually exclusive, meaning they can't happen at the same time, P(A and B) becomes zero. The chance you'll go to a Chinese restaurant <u>or</u> an Italian restaurant is the chance you'll go to a Chinese restaurant + chance you'll go to an Italian restaurant. P(C) + P(I).

iii. Adding the probability of all possible outcomes results in a sum of 1. There's a 100% chance one of the possible results will occur.

II. Tree Diagram 1

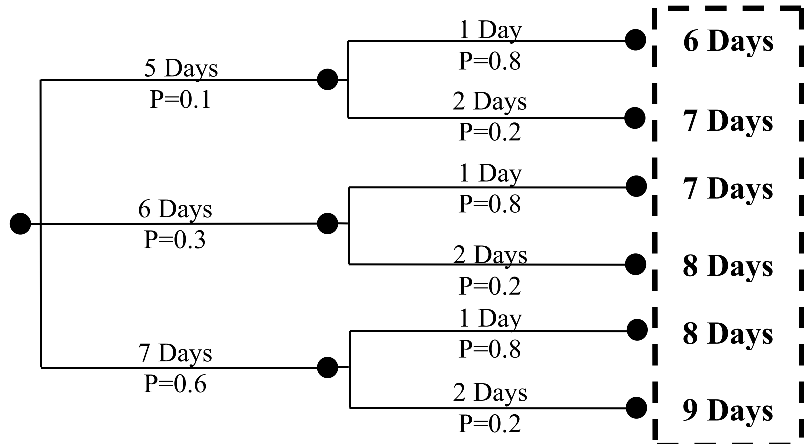
a. Business decisions often come with uncertainty on many levels. One way to capture all what's going on is with a tree diagram—a graphical representation of all possible outcomes.

b. Consider a delivery company estimating how long it takes to deliver a package. Each package goes through two stages: national shipping, where it's flown around the country, and local delivery.

i. Suppose national shipping can take 5, 6 or 7 days with a probability of 10%, 30%, and 60%, respectively.

ii. Also suppose that local delivery takes 1 or 2 days, with a probability of 80% and 20%, respectively.

c. Our tree diagram "maps" every path the delivery can take, with probabilities indicated. The total time for delivery is in the box on the far right.



i. Note each small branch of the tree are identical to each other; that tells us that probabilities are independent. It doesn't matter how long the national shipping takes—that won't effect how long the local delivery takes.

d. The delivery will take anywhere from 6 to 9 days. But some time frames are more likely than others.

i. How likely is the first scenario? National shipping is super-fast **AND** local delivery is super-fast. So we multiply: 0.1*0.8 = 0.08.

ii. Let's do the same thing for all scenarios:

Scenario	Total Time	National Probability	Local Probability	Overall Probability
1	6 Days	0.1	0.8	0.08
2	7 Days	0.1	0.2	0.02
3	7 Days	0.3	0.8	0.24
4	8 Days	0.3	0.2	0.06
5	8 Days	0.6	0.8	0.48
6	9 Days	0.6	0.2	0.12

e. While it looks like we have six different scenarios, for purposes of estimating project completion, we really only have four: the delivery will take 6 days, 7 days, 8 days, or 9 days.

i. There are two ways for it to take 7 days: scenario #2 OR scenario #3. And two ways for 8 days: scenario #4 OR scenario #5.

ii. Since it can't take 1 and 2 days for local delivery, we can say these scenarios are mutually exclusive. Thus we can add probabilities:

Total Time	Scenario Probabilities		Overall Probability
6 days	0.08		0.08
7 days	0.02	0.24	0.26
8 days	0.06	0.48	0.54
9 days	0.12		0.12

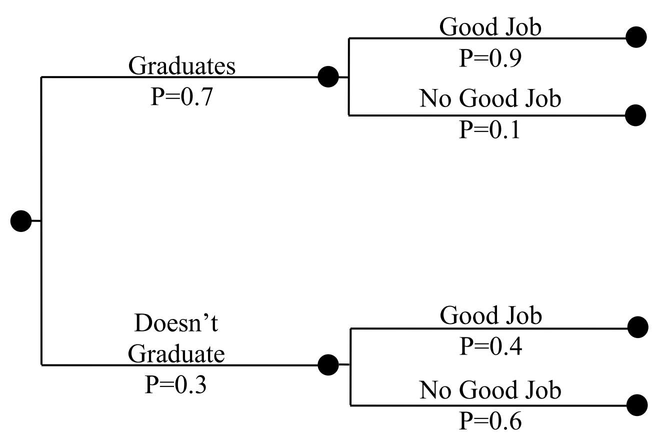
I. Tree Diagram 2

a. What's nice about tree diagrams is that they can handle dependent probabilities just fine—just make the lower parts different from the upper parts.

b. Suppose Amir is majoring in economics. He wants to know how likely he'll have a good job.

   i. Suppose Amir has an 70% chance of graduating.

      1. If he graduates with a major in economics, suppose he has a 90% chance of getting a good job.
      2. If he doesn't pass college, suppose he has a 40% chance of getting a good job.

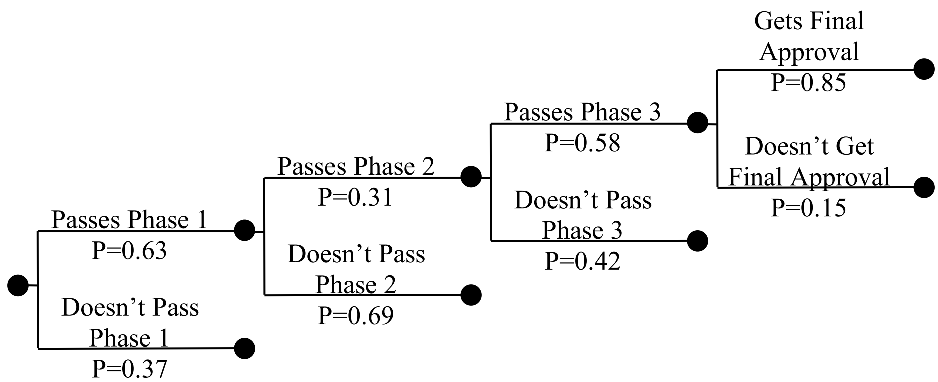


   ii. Now the branches are different because the chance of getting a good job changes if he graduates vs if he doesn't graduate.

II. Tree Diagram 3

a. There's a lot of variety possible here. Consider drug development. The FDA has three phases of drug testing to ensure that the drug is safe and effective, plus a final approval phase. If the drug fails any phase, it can't be approved. (Contrast this with Amir, who can get a job even if he doesn't graduate college.)

b. So we can build the tree diagrams using probabilities from [this article] about the success rates in different phases:



c. We can find the overall chance of success by multiplying all the probabilities of each stage together and get 0.096, or 9.6% that a drug will be approved by the FDA.
    i. Note this is an overall probability; cancer drugs are less likely to be approved and treatments for blood-related conditions are most likely to be approved.